Sub-diffraction Imaging using Fourier Ptychography and Structured Sparsity

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Diffraction Imaging

Why Ptychography?

- Problems of focus:

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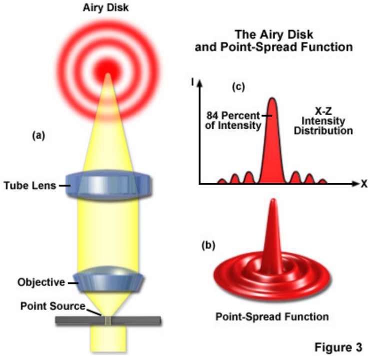


Figure: Microscopic imaging setup.

Image source: http://zeiss-campus.magnet.fsu.edu/articles/basics/resolution.html

Resolution limit

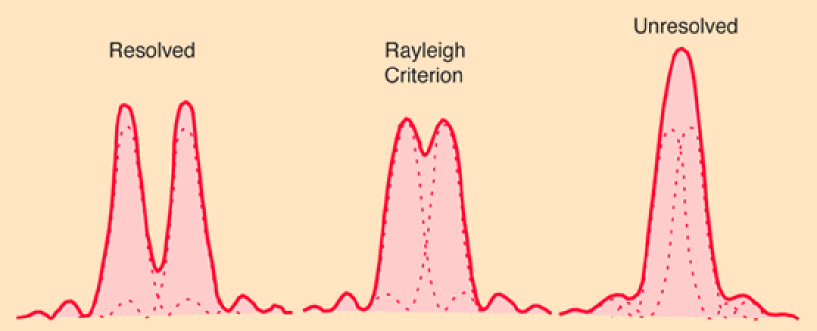


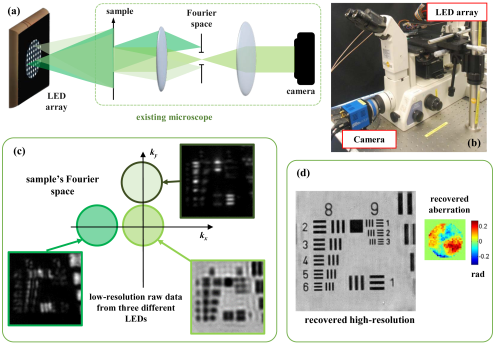
Figure: Resolving two point sources.

$$\text{Diffraction spot size} \propto \frac{\text{distance of object from lens}}{\text{aperture of imaging lens}}.$$

Image source: http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/Raylei.html

Fourier Ptychography Setup

Short-distance imaging



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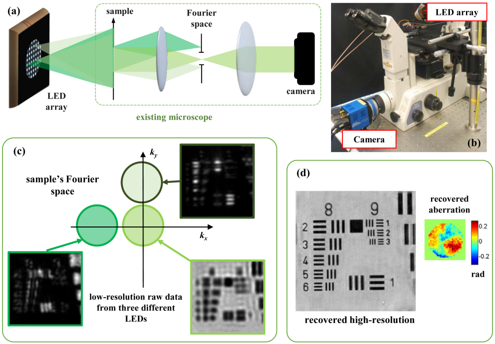

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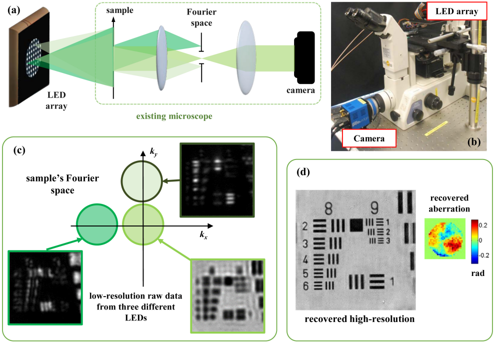

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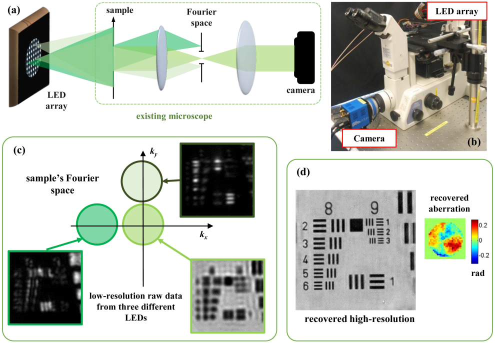

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- Diffraction information is collected from overlapping iluminated regions on an object, effectively giving large synthetic aperture.
- Optical sensors can only detect magnitude.
  - Phase information is lost. $\implies$ Requires a reconstruction algorithm to estimate phase!

Fourier Ptychography Setup

Long-distance imaging

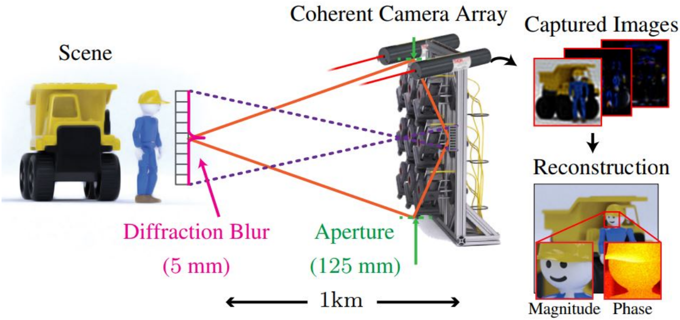

Figure: Object is imaged by using an "overlapping" camera array, generating large synthetic aperture [Holloway et. al, '16].

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- At the cost of:
  - Higher number of measurements (sample complexity).
  - Added post-processing time for the recovery algorithm (running time complexity).

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Standard setup

- Signal (vectorized image frame):

$$\mathbf{x} \in \mathbb{C}^n$$

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Equivalently,

$$\mathbf{y} = |\mathcal{A}(\mathbf{x})| = [\mathbf{y}_1^\top \ldots \mathbf{y}_i^\top \ldots \mathbf{y}_N^\top],$$

$$\text{where} \quad \mathcal{A} = [\mathcal{A}_1^\top \ldots \mathcal{A}_i^\top \ldots \mathcal{A}_N{}^\top],$$

and $\mathbf{y} \in \mathbb{R}^{nN}$ and $\mathcal{A} : \mathbb{C}^n \to \mathbb{C}^{nN}$, with $m = nN \gg n$.

Flow of optical operations

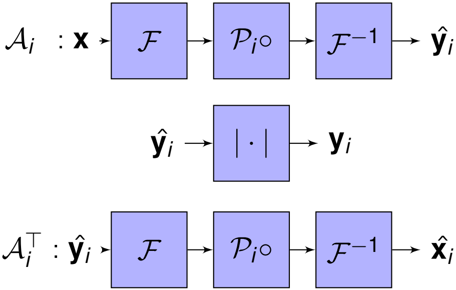

Figure: Sampling procedure, using operator $\mathcal{A}_i$ in conventional Fourier ptychographic setups. Camera index is denoted by $i = [N]$.

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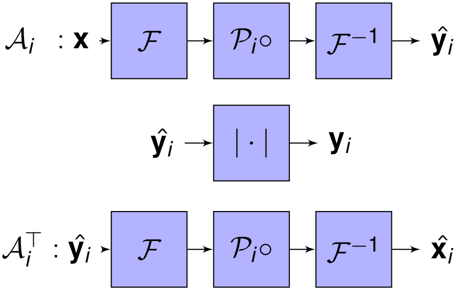

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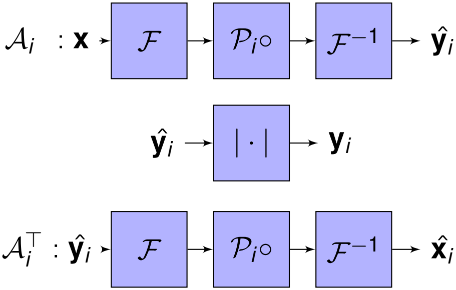

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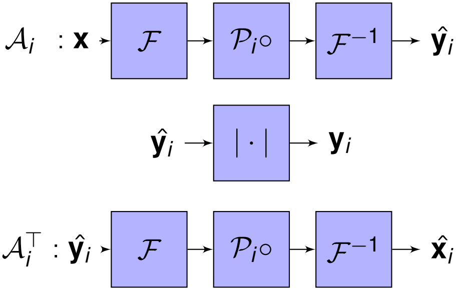

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- $\mathcal{P}_i$ is a pupil mask (bandpass filter),
- $\mathcal{P}_i$'s cover different parts of the Fourier domain image ($\circ$ is Hadamard product).

Standard phase retrieval problem:

Observation Model

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$$\mathbf{y} = |\mathcal{A}(\mathbf{x})|, \qquad \mathcal{A} : \mathbb{C}^n \to \mathbb{C}^m, \quad m \gg n$$

Goal: Recover **x** from **y.**

(Statistical)

How many measurements do we need for stable recovery?

(Computational)

How quickly can we perform the recovery?

What is known

$$\mathbf{y} = |\mathcal{A}(\mathbf{x})|, \qquad \mathcal{A} : \mathbb{R}^n \to \mathbb{R}^m, \;\; m > n$$

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Solution methodology involves estimating phase and signal information in alternating steps [Gerschberg-Saxton ’72, Fienup ’78].

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Challenges:

- High sample complexity ($\mathcal{O}(n)$ measurements; can be huge if n is large).
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- High sample complexity ($\mathcal{O}(n)$ measurements; can be huge if n is large).
- High running time; algorithms are not scalable.

Solution:

- Utilize inherent structure in the signal! Most images to be acquired have *underlying (structured) sparsity*!

New goal: Recover s-sparse signal $\mathbf{x}$ from magnitude-only ptychographic measurements $\mathbf{y}$.

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Given:

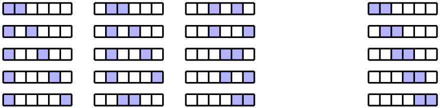
$$\mathbf{y} = |\mathcal{A}(\mathbf{x})|, \qquad \mathcal{A} : \mathbb{R}^n \to \mathbb{R}^m, \quad m \ll nN$$

Recover: $\mathbf{x}$, such that $\|\mathbf{x}\|_0 \leq s$.

Is sparsity the only prior that can be used?

Modeling Sparsity

- Block/group sparsity (this paper).



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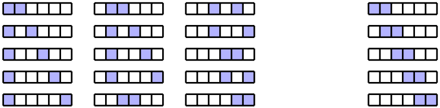
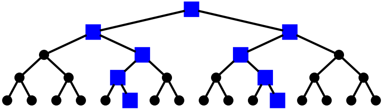


- Tree sparsity.



Our contributions

Sub-diffraction Imaging using Fourier Ptychography and Structured Sparsity

1. Suitable *sub-sampling* strategies for Fourier ptychography.
   - Reduces the number of samples acquired for image reconstruction.

2. New (structured) sparsity-based algorithms for solving the Fourier ptychographic phase retrieval problem.

Contributions (I) : Sub-sampling Strategies

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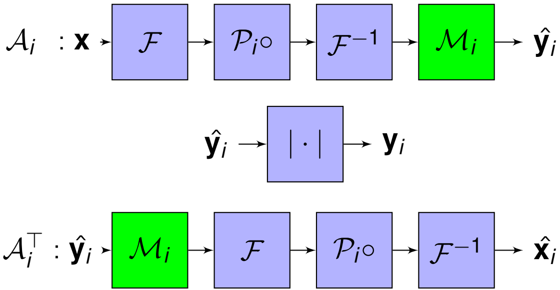

Figure: Sampling operator $\mathcal{A}_i$. The green box is extra subsampling step.

$$\mathcal{A}_i = \mathcal{M}_i \mathcal{F}^{-1} \mathcal{P}_i \circ \mathcal{F} \quad \text{and} \quad \mathcal{A}_i^\top = \mathcal{F}^{-1} \mathcal{P}_i \circ \mathcal{F} \mathcal{M}_i,$$

- The sub-sampling masks $\mathcal{M}_i$ resembles the operation of an *identity*, in the conventional setup (i.e. all measurements are retained).

Contributions (I) : Sub-sampling Strategies

Uniform Random Pixel Patterns

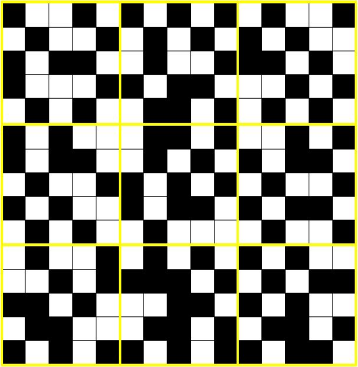

Figure: $N = 9$ camera grid.

- Masking elements of $\mathcal{M}_i$ are picked according to independent standard uniform random variables u_j^i.
- Total of $m = f \times (nN)$ measurements are retained, from all N cameras, where f denotes the fraction of samples (or pixels).
- For an input vector $\mathbf{v} \in \mathbb{C}^n$, the sub-sampling mask operates as

$$\mathcal{M}_i(\mathbf{v})_j = \begin{cases} 0 & u_j^i > f, \\ v_j & u_j^i \leq f. \end{cases}$$

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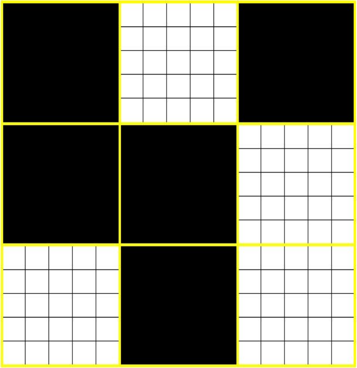

Figure: $N = 9$ camera grid.

- Turn some cameras “on" or “off".
- Masking elements of $\mathcal{M}_i$ are picked up according to continuous *standard uniform* variables u_i.
- For a vector input $\mathbf{v} \in \mathbb{C}^n$, the sub-sampling mask,

$$\mathcal{M}_i(\mathbf{v}) = \begin{cases} \mathbf{0} & u_i > f, \\ \mathbf{v} & u_i < f. \end{cases}$$

Contributions (II) : Sparse signal and phase recovery

The signal estimate can be posed as the solution to the non-convex optimization problem:

$$\min_{\mathbf{x}} \sum_{i=1}^{N} \| |\mathcal{A}_i(\mathbf{x})| - \mathbf{y}_i \|_2^2, \quad \text{s.t. } \mathbf{x} \in \mathfrak{M}_s^b,$$

- $\mathbf{x}$ is the signal in the sparse domain,
- $\mathfrak{M}_s^b$ denotes the model of the signal, consisting of a set of s-sparse signals with uniform block length $b \in \mathbb{Z}$.
- $\mathcal{A}$ is modified measurement operator, accounts for the domain transformation *and sub-sampling mask*.

*For the standard sparse model $b = 1$; for the block sparse model $b > 1$.

Contributions (II) : CoPRAM Framework

Adaptation for Fourier ptychography

Utilize the CoPRAM (Compressive Phase Retrieval with Alternating Minimization) framework [Jagatap, Hegde '17]:

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For $t = 0, \ldots, T$:

- Phase estimation: $\mathbf{P}^t = \text{diag}\left(\text{sign}\left(\mathcal{A}(\mathbf{x}^t)\right)\right)$.
- Signal estimation: $\mathbf{x}^t = \text{argmin}_{\mathbf{x}' \in \mathfrak{M}_s^b} \left\|\mathcal{A}(\mathbf{x}') - \mathbf{P}^t\mathbf{y}\right\|_2$.

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Key features:

- Utilizes Model-based CoSaMP [Baraniuk et. al. '10] to recover (structured) sparse signal estimate $\mathbf{x}^t$ $\implies$ reduced sample complexity.

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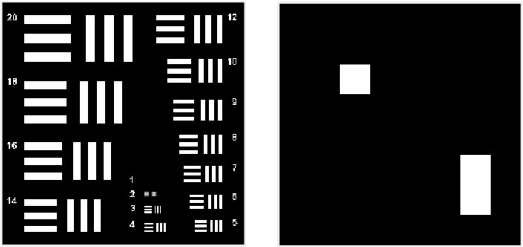
- Initialization strategy for *faster* convergence.
- No tuning parameters!

Experimental validation

Ground truth

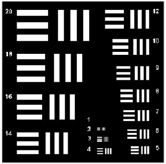


(a) (b)

Figure: (a) Resolution chart, used as ground truth (b) simulated block sparse image, used as ground truth for experimental analysis.

Simulation Results

Random pixel sub-sampling



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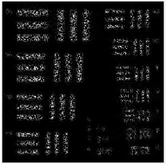
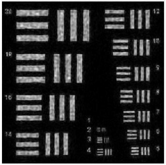
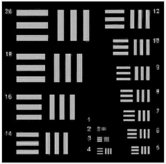


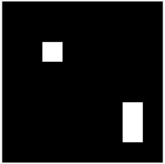
(b)
Initial center,
SSIM=0.3517



(c)
AltMin,
SSIM=0.3369



(d)
CoPRAM,
SSIM=0.8740



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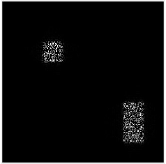
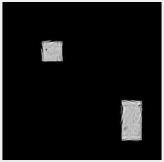
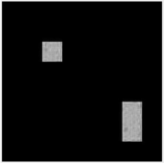


(b)
Initial center,
SSIM=0.9969



(c)
CoPRAM,
SSIM=0.99995



(d)
Block CoPRAM,
SSIM=0.99998

Figure: Sub-sampling ratio $f = m/nN = 0.3$, assumed sparsity $s = 0.25n$ (top) and $s = 0.1n$ (bottom) both in spatial domain.

Simulation results

Phase transitions

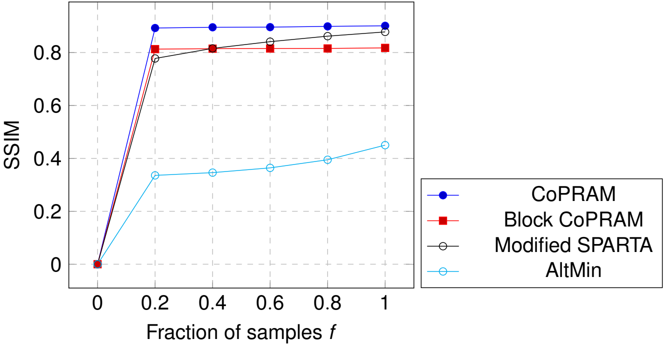
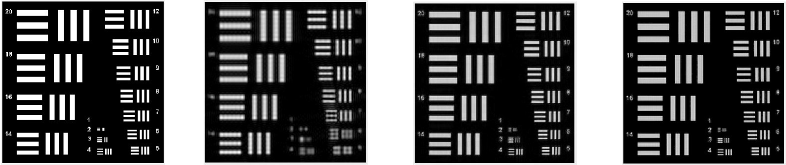


Figure: Variation of SSIM with sub-sampling ratio $f = m/nN$, with (spatial) sparsity $s = 0.25n$, (block size $b = 4 \times 4$ for Block CoPRAM), for the Resolution Chart image.

Simulation Results

Random camera sub-sampling



(a) Ground truth | (b) Initial center, SSIM=0.3927 | (c) AltMin, SSIM=0.4225 | (d) CoPRAM, SSIM=0.9053

Figure: (a) Ground truth (b) center image, reconstruction from 50% camera measurements using (c) AltMin (d) CoPRAM, assuming sparsity $s = 0.25n$ in spatial domain.

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Open questions:

- Theoretical guarantees on convergence.
- Extension to other models of sparsity.

Questions?

Interested in knowing more?
Check our project website:

https://gaurijagatap.github.io/Sparse-image-super-resolution/