Model Corrected Low Rank Ptychography

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Problem Setup

Objective: To design an algorithm for Fourier ptychographic imaging of dynamic, time-varying targets.

Main Challenges:

- Large number of samples required for frame-by-frame recovery.
- Real videos are usually, *approximately* low rank; imposing an exact low-rankness can reduce accuracy of reconstruction.

Our Contribution

We design a low-rank ptychography algorithm that:

- Efficiently models *low-rankness* of videos for improved sample complexity as compared to existing "single-frame" methods.
- Works with novel "under-sampling" strategies from [CJNHV18] that can be easily incorporated into existing Fourier ptychographic setups.

Concept of Fourier Ptychography

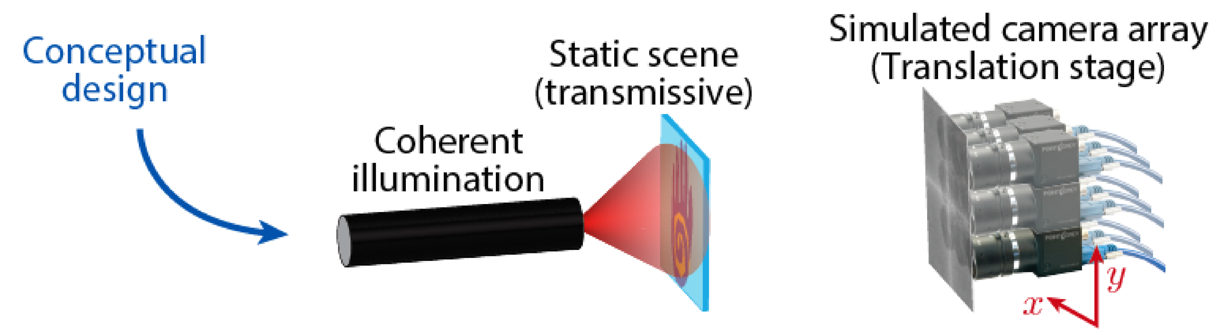

Figure 1: Conceptual design for a fourier ptychography system[1].

1. J. Holloway et.al. ,"Toward Long-distance Subdiffraction Imaging Using Coherent Camera Arrays", IEEE TCI, '16.

Data Acquisition Setup and Under-sampling Strategies

Recover video matrix $\mathbf{X} := [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_q]$, $\mathbf{X} \in \mathbb{R}^{n \times q}$, using measurement operator $\mathcal{A}_{i,k}$, where $i \in [1, \ldots, N]$ (camera index) and $k \in [1, \ldots, q]$ (video frame index) from measurements $\mathbf{y}_{i,k}$,

$$\mathbf{y}_{i,k} = |\mathcal{A}_{i,k}(\mathbf{x}_k)|$$
$$\mathcal{A}_{i,k}(\cdot) = \mathcal{M}_{i,k}\mathcal{F}^{-1}\mathcal{P}_i \circ \mathcal{F}(\cdot)$$

We assume that the rank of the true matrix $\mathbf{X}^*$ is no greater than r.

Under-sampling schemes:

Figure 2: For 3×3 camera grid ($N = 9$) (left) pixel-wise under-sampling; (right) camera under-sampling

Flow of Mearsurement

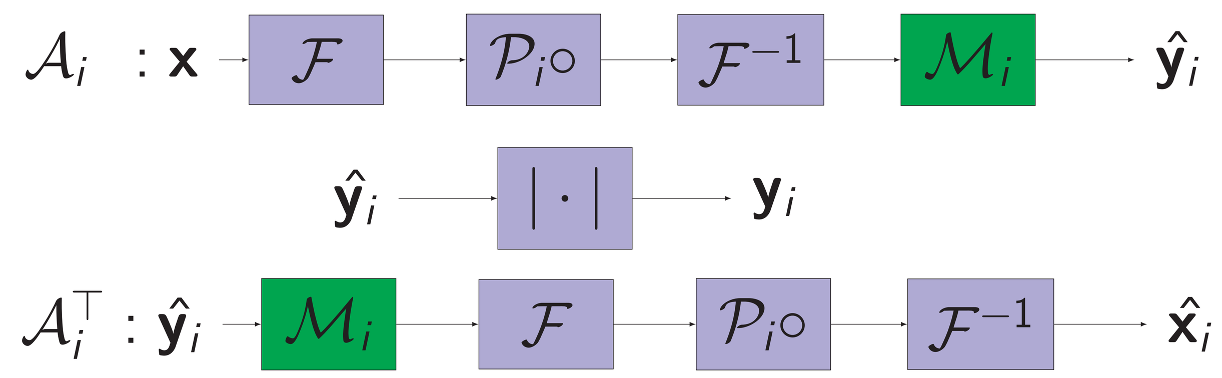

Figure 3: Sequence of operations defined by $\mathcal{A}_i$.

(MLR-Ptych: Stage I) Low-rank reconstruction

Model corrected Low Rank Ptychography:
"Slowly changing" video assumption: first few (r) singular values of $\mathbf{X}^*$ are much greater than remaining. Then, we recover $\mathbf{X}$ as the solution to the non-convex problem:

$$\underset{\mathbf{X}}{\operatorname{argmin}} \sum_{k=1}^{q} \sum_{i=1}^{N} \| \mathbf{y}_{i,k} - |\mathcal{A}_{i,k}(\mathbf{x}_k)| \|_2^2, \quad \text{s.t.} \quad \operatorname{rank}(\mathbf{X}) = r$$

Solution methodology:

- Adapt *low-rank phase retrieval* algorithm [VNE17].
- Rank-r matrix $\mathbf{X}^*$ can be written as $\mathbf{X}^* = \mathbf{U}_{n \times r}\mathbf{B}_{r \times q}$, where $\mathbf{U}, \mathbf{V}$ have mutually orthonormal columns.

(MLR-Ptych: Stage II) Model correction

If real video is not low-rank, do "model-correction":

$$\hat{\mathbf{X}} := \tilde{\mathbf{X}} + \operatorname{argmin}_{\mathbf{E}} \sum_{k=1}^{q} \sum_{i=1}^{N} \| \mathbf{y}_{i,k} - |\mathcal{A}_{i,k}(\mathbf{x}_k + \mathbf{e}_k)| \|_2^2$$

where $\mathbf{E} = [\mathbf{e_1}, \mathbf{e_2}, \ldots \mathbf{e_q}]$, $E \in \mathbb{R}^{n \times q}$ is the modeling error.

Algorithm: MLR-Ptych: Stage I

Initialization:

- $\mathbf{x}_k^0 \leftarrow \sqrt{\frac{1}{N}\sum_{i=1}^{N} \mathbf{y}_{i,k}^2}$ for $k = 1, \ldots, q$
- $[\mathbf{U}^0, \mathbf{S}^0, \mathbf{V}^0] \leftarrow SVD(\mathbf{X}^0)$, and, $\mathbf{b}_k^0 \leftarrow (\mathbf{S}^0\mathbf{V}^{0\top})_k$, $\forall k$

Descent: Use $\mathbf{U}^0$ and $\mathbf{b}_k^0$ as initialization. Iterate for $t = 1, \ldots T$:

- $\mathbf{C}_k^t \leftarrow \operatorname{diag}(phase(\mathcal{A}_k(\mathbf{U}^{t-1}\mathbf{b}_k^{t-1})))$, $k = 1, \ldots, q$
- $\mathbf{U}^{tmp} \leftarrow \operatorname{argmin}_{\tilde{\mathbf{U}}} \sum_k \left\| \mathbf{C}_k^t \mathbf{y}_k - \mathcal{A}_k(\tilde{\mathbf{U}}\mathbf{b}_k^{t-1}) \right\|^2$,
- $\mathbf{U}^t \leftarrow QR(\mathbf{U}^{tmp})$
- $\mathbf{b}_k^t \leftarrow \operatorname{argmin}_{\tilde{\mathbf{b}}_k} \left\| \mathbf{C}_k^t \mathbf{y}_k - \mathcal{A}_k(\mathbf{U}^t\tilde{\mathbf{b}}_k) \right\|^2$, $k = 1, \ldots, q$

Algorithm: MLR-Ptych: Stage II

Model correction: Initialize $\hat{\mathbf{x}}_k^0 = \mathbf{U}^T\mathbf{b}_k^T$ for each $k = 1, \ldots, q$, Iterate over $t = 1, 2, \ldots, T'$:

- $\mathbf{C}_k^t \leftarrow \operatorname{diag}(phase(\mathcal{A}_k(\hat{\mathbf{x}}_k^{t-1})))$, $\forall k$
- $\mathbf{e}_k^t \leftarrow \operatorname{argmin}_{\mathbf{e}}(\|\mathbf{C}_k^t\mathbf{y}_k - \mathcal{A}_k(\hat{\mathbf{x}}_k^{t-1} + \mathbf{e})\|_2^2 + \tau\|\mathbf{e}\|_2^2)$, $\forall k$
- $\hat{\mathbf{x}}_k^t = \hat{\mathbf{x}}_k^{t-1} + \mathbf{e}_k^t$, $\forall k$

Output: $\hat{\mathbf{X}}^{T'}$.

Results

Pixel-wise random under-sampling

Comparison for frame 66 of video "fish", using 50% pixels.

Random camera under-sampling

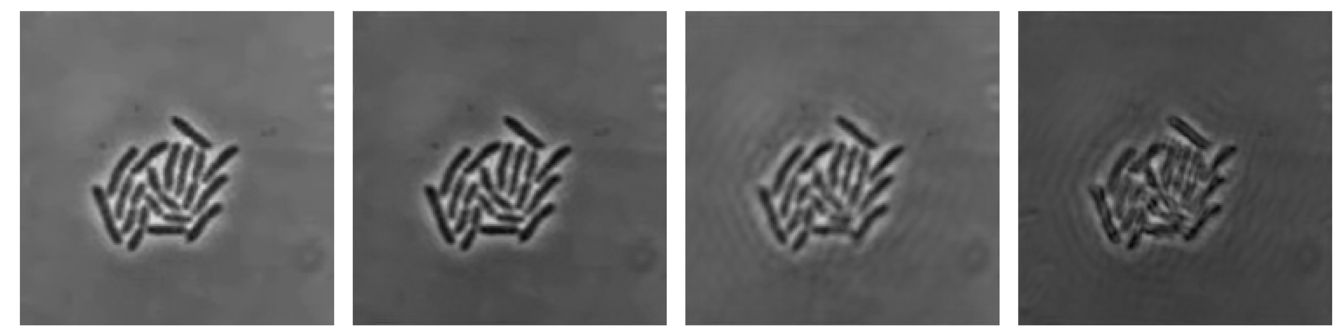

Figure 4: Comparison for frame 66 of video "bacteria", using 50% cameras.

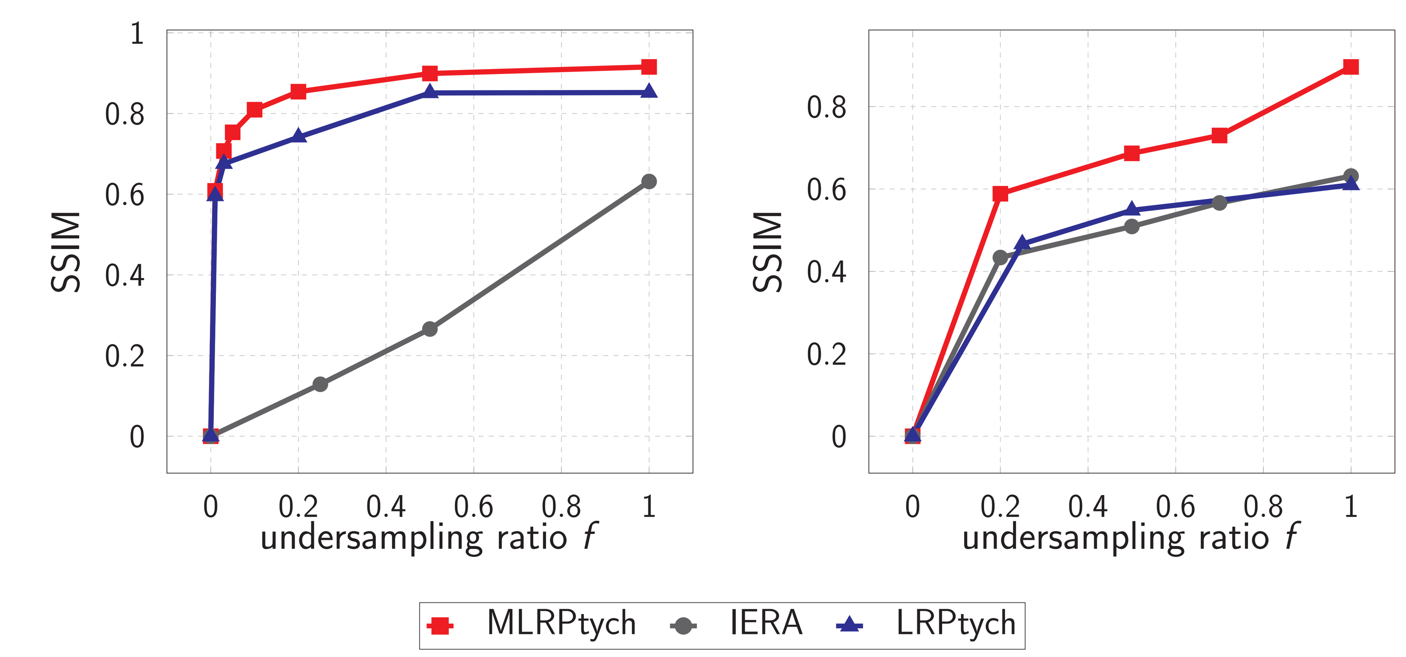

Figure 5: Comparison on different under sample ratios using fraction of (left) *pixels* and (right) *cameras* at random, for the video "dog".

References

[VNE17] N. Vaswani, S. Nayer and Y. Eldar. "Low Rank Phase Retrieval", IEEE Trans. Signal Processing, 2017.

[CJNHV18] Z. Chen, G. Jagatap, S. Nayer, C. Hegde and N. Vaswani, "Low Rank Fourier Ptychography." IEEE ICASSP, 2018.