

Towards Sample-Optimal Methods for Solving Random Quadratic Equations with Structure

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Compressed Sensing Fr-AM-1-5
International Symposium on Information Theory (ISIT), 2018

Problem Setup

Random Quadratic Equations

- ▶ Unknown vector of parameters, $\mathbf{x}^* \in \mathbb{R}^n$
- ▶ Observations $\mathbf{y} \in \mathbb{R}^m$ of the form:

$$y_i = |\langle \mathbf{a}_i, \mathbf{x}^* \rangle|^p, \quad i = [m], \quad \text{s.t. } \mathbf{x}^* \in \mathcal{M}_s$$

- ▶ $\mathcal{M}_s \subset \mathbb{R}^n$ is a *model* set that reflects the structural constraints on \mathbf{x}^* .
- ▶ Under-determined Gaussian observations, $\mathcal{A} = [\mathbf{a}_1 \dots \mathbf{a}_i \dots \mathbf{a}_m]^\top \in \mathbb{R}^{m \times n}$ with $m < n$.

Task: Estimate \mathbf{x}^* from either absolute-value ($p = 1$) or quadratic ($p = 2$) measurements \mathbf{y} .

Applications

- ▶ Phase retrieval.
 - ▶ Fourier imaging.
 - ▶ Sub-diffraction imaging (eg. Ptychography).

- ▶ Polynomial neural networks with quadratic activation functions.

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Observation Model

Model: $\mathbf{x} \in \mathbb{C}^n$

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Goal: Recover \mathbf{x} from \mathbf{y} .

(Statistical)

How many measurements do we need for stable recovery?

(Computational)

How quickly can we perform the recovery?

What is known

$$\mathbf{y} = |\mathcal{A}(\mathbf{x})|, \quad \mathcal{A} : \mathbb{R}^n \rightarrow \mathbb{R}^m, \quad m > n$$

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Popular solution methodology involves estimating phase information $\text{phase}(\mathcal{A}(\mathbf{x}^{t-1}))$ and linearized signal information \mathbf{x}^t in alternating steps [Gerschberg-Saxton '72, Fienup '78].

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Challenges:

- ▶ High sample complexity ($\mathcal{O}(n)$ measurements for Gaussian operator \mathcal{A} ; can be huge if n is large).
- ▶ High running time; algorithms are not scalable.

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Solution:

- ▶ Utilize inherent structure of \mathbf{x} !
 - ▶ Most images to be acquired have *underlying (structured) sparsity!*
 - ▶ Weights of teacher network to be learned can be sparse.

Sparsity

Phase Retrieval via Alternating Minimization

New goal: Recover s -sparse signal \mathbf{x} from magnitude-only Gaussian measurements \mathbf{y} .

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Given:

$$\mathbf{y} = |\mathcal{A}(\mathbf{x})|, \quad \mathcal{A} : \mathbb{R}^n \rightarrow \mathbb{R}^m, \quad m \ll n$$

Recover: \mathbf{x} , such that $\|\mathbf{x}\|_0 \leq s$.

Sample complexity

Algorithm	Initialization Sample complexity	Convergence Sample complexity	Running time	Assumptions
AltMin	$\mathcal{O}(n \log^3 n)$	$\mathcal{O}(n)$	$\mathcal{O}(n^2 \log^3 n)$	none
AltMinSparse	$\mathcal{O}(s^2 \log^3 n)$	$\mathcal{O}(s)$	$\mathcal{O}(s^2 n \log n)$	s -sparse $x_{\min}^* \approx \frac{c}{\sqrt{s}} \ \mathbf{x}^*\ _2$
ℓ_1 -PhaseLift	–	$\mathcal{O}(s^2 \log n)$	$\mathcal{O}\left(\frac{n^3}{\epsilon^2}\right)$	s -sparse
SPARTA	$\mathcal{O}(s^2 \log n)$	$\mathcal{O}(s \log n/s)$	$\mathcal{O}(s^2 n \log n)$	s -sparse $x_{\min}^* \approx \frac{c}{\sqrt{s}} \ \mathbf{x}^*\ _2$
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Is sparsity the only prior that can be used?

Modeling Sparsity

- ▶ Block/group sparsity.

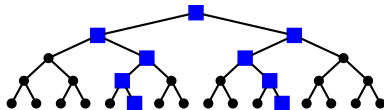


Modeling Sparsity

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- ▶ Tree sparsity.



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CoPRAM	$\mathcal{O}(s^2 \log n)$	$\mathcal{O}(s \log n/s)$	s-sparse
Model-based CoPRAM	??	$\mathcal{O}(s + \log(\text{card}(\mathbb{M}_{4s})))$	model sparse \mathcal{M}_s
Block CoPRAM	$\mathcal{O}(s^2/b \log n)$	$\mathcal{O}(s + (s/b) \log n)$	model sparse \mathcal{M}_s^b s-sparse, block length b
Tree CoPRAM	??	$\mathcal{O}(s)$	model sparse \mathcal{M}_s s-tree sparse

Our contributions

Sample-optimal methods

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1. New algorithmic framework for *any* given model-based sparsity constraint.
2. Theoretical guarantees on convergence.
3. Novel initialization strategy.

Contributions (I) : Sparse signal and phase recovery

The signal estimate can be posed as the solution to the non-convex optimization problem:

$$\min_{\mathbf{x} \in \mathcal{M}_s, \mathbf{p} \in \mathcal{P}} \|\mathbf{A}\mathbf{x} - \mathbf{p} \circ \mathbf{y}\|_2$$

- ▶ $\mathbf{x} \in \mathbb{R}^n$ is a sparse signal (or weight vector),
- ▶ Let \mathbb{M}_s denote the set of all allowable supports $\{S_1 \dots S_i \dots S_N\}$, such that $|S_i| \leq s$, then $\mathcal{M}_s = \{\mathbf{x} \in \mathbb{R}^n \mid \text{supp}(\mathbf{x}) \in \mathbb{M}_s\}$,
- ▶ $\mathbf{A} \in \mathbb{R}^{m \times n}$ is a measurement operator with $a_{ij} \sim \mathcal{N}(0, 1)$,
- ▶ $\mathbf{p} \in \mathbb{R}^m$ is a vector that stores the missing sign information with entries constrained to be in $\{-1, 1\}$ ($:= \mathcal{P}$),
- ▶ $\mathbf{y} \in \mathbb{R}^m$ are observations or labels.

Contributions (I) : Model-based CoPRAM Framework

Utilize the CoPRAM (Compressive Phase Retrieval with Alternating Minimization) framework [Jagatap, Hegde '17]:

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- ▶ Compute marginals: $\text{diag}(\mathbf{M}) := M_{jj} = \frac{1}{m} \sum_{i=1}^m y_i^2 \mathbf{a}_{ij}^2$ for $j = [n]$.
- ▶ Set: $\hat{\mathcal{S}} \leftarrow \text{MODELAPPROX}(\text{diag}(\mathbf{M}))$.
- ▶ $\mathbf{v} \in \mathbb{R}^n \leftarrow$ top s.v. of $\mathbf{M}_{\hat{\mathcal{S}}} = \frac{1}{m} \sum_{i=1}^m y_i^2 \mathbf{a}_{i\hat{\mathcal{S}}} \mathbf{a}_{i\hat{\mathcal{S}}}^T$ for $\hat{\mathcal{S}}$, and $\mathbf{0}$ for $\hat{\mathcal{S}}^c$.
- ▶ Compute: $\mathbf{x}^0 \leftarrow \phi \mathbf{v}$, with $\phi^2 = \frac{1}{m} \sum_{i=1}^m y_i^2$.

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For $t = 0, \dots, T$, alternate:

- ▶ Phase estimation: $\mathbf{p}^t = \text{sign}(\mathbf{A}\mathbf{x}^t)$.
- ▶ Signal estimation: $\mathbf{x}^t = \text{argmin}_{\mathbf{x}' \in \mathcal{M}_s} \|\mathbf{A}\mathbf{x}' - \mathbf{p}^t \circ \mathbf{y}\|_2$.

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 \implies reduced sample complexity.
- ▶ Initialization strategy for *faster* convergence.
- ▶ No tuning parameters!

Contributions (II) - Convergence guarantees

Theorem

Given an initialization $\mathbf{x}^0 \in \mathcal{M}_s$ satisfying $\text{dist}(\mathbf{x}^0, \mathbf{x}^) \leq \delta_0 \|\mathbf{x}^*\|_2$, for $0 < \delta_0 < 1$, if we have number of Gaussian measurements,*

$$m > C(s + \log(\text{card}(\mathbb{M}_{4s}))),$$

then the iterates \mathbf{x}^{t+1} of model-based CoPRAM satisfy:

$$\text{dist}(\mathbf{x}^{t+1}, \mathbf{x}^*) \leq \rho_0 \text{dist}(\mathbf{x}^t, \mathbf{x}^*),$$

where $\mathbf{x}^t, \mathbf{x}^{t+1}, \mathbf{x}^ \in \mathcal{M}_s$, and $0 < \rho_0 < 1$ is a constant, with probability greater than $1 - e^{-\gamma m}$, for positive constant γ .*

Contributions (II) - Convergence guarantees

Proof outline

- ▶ Per-iteration error for the t^{th} iteration of model-based CoPRAM, with L iterations of model-based CoSaMP:

$$\|\mathbf{x}^{t+1} - \mathbf{x}^*\|_2 \leq (\rho_1 \rho_3)^L \|\mathbf{x}^* - \mathbf{x}^t\|_2 + \frac{(\rho_1 \rho_4 + \rho_2)}{(1 - \rho_1 \rho_3)} \|E_{ph}\|_2$$

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- ▶ $\rho_1, \rho_2, \rho_3, \rho_4$ are appropriate constants, and E_{ph} is the error in estimating phase in the t^{th} run of Model-based CoPRAM.
- ▶ Bound the phase error term $\|E_{ph}\|_2$ via Lemma III.2 as:

$$\|E_{ph}\|_2^2 = \frac{4}{m} \sum_{i=1}^m (\mathbf{a}_i^\top \mathbf{x}^*)^2 \cdot \mathbf{1}_{\{\text{sign}(\mathbf{a}_i \mathbf{x}^t) \text{sign}(\mathbf{a}_i \mathbf{x}^*) = -1\}} < \rho_5^2 \|\mathbf{x}^t - \mathbf{x}^*\|_2^2.$$

- ▶ Per-step error reduction scheme of the form:

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- ▶ Convergence criterion of the form (for $\rho_0 < 1$):

$$\text{dist} \left(\mathbf{x}^{t+1}, \mathbf{x}^* \right) \leq \rho_0 \text{dist} \left(\mathbf{x}^t, \mathbf{x}^* \right).$$

as long as \mathbf{x}^0 satisfies $\text{dist} \left(\mathbf{x}^0, \mathbf{x}^* \right) \leq \delta_0 \left\| \mathbf{x}^* \right\|_2$,

- ▶ $\text{dist} \left(\mathbf{x}_1, \mathbf{x}_2 \right) := \min \left\{ \left\| \mathbf{x}_1 - \mathbf{x}_2 \right\|_2, \left\| \mathbf{x}_1 + \mathbf{x}_2 \right\|_2 \right\}$.

Contributions (II) - Convergence guarantees

Key Lemma

Lemma

(Lemma III.2) As long as the initial estimate is a small distance away from the true signal $\mathbf{x}^ \in \mathcal{M}_s$, $\text{dist}(\mathbf{x}^0, \mathbf{x}^*) \leq \delta_0 \|\mathbf{x}^*\|_2$, and subsequently, $\text{dist}(\mathbf{x}^t, \mathbf{x}^*) \leq \delta_0 \|\mathbf{x}^*\|_2$, where \mathbf{x}^t is the t^{th} update of model-based CoPRAM, then the following bound holds,*

$$\frac{4}{m} \sum_{i=1}^m \left(\mathbf{a}_i^\top \mathbf{x}^* \right)^2 \cdot \mathbf{1}_{\{(\mathbf{a}_i^\top \mathbf{x}^t)(\mathbf{a}_i^\top \mathbf{x}^*) \leq 0\}} \leq \rho_5^2 \|\mathbf{x}^t - \mathbf{x}^*\|_2^2,$$

with probability greater than $1 - e^{-\gamma_2 m}$, where γ_2 is a positive constant, as long as $m > C(s + \log(\text{card}(\mathbb{M}_{4s})))$ and $\rho_5^2(\delta_0) < 1$.

Contributions (II) - Convergence guarantees

Corollary for tree sparse signals

Corollary

As a consequence of Theorem 1, if \mathcal{M}_s is a model representing rooted tree sparse signals with sparsity s , then Tree CoPRAM is linearly convergent and requires a Gaussian sample complexity of $m > Cs$, as long as the initialization \mathbf{x}^0 satisfies $\text{dist}(\mathbf{x}^0, \mathbf{x}^) \leq \delta_0 \|\mathbf{x}^*\|_2$.*

- ▶ $m = O(s)$ samples are necessary for reconstructing any s -sparse parameter vector even in the linear case (where perfect phase information is available).
- ▶ Implies information-theoretic optimality (up to constants) of our proposed approach.

Contributions (III) - Initialization Strategy

- ▶ Spectral initialization [Wang et al '17, Jagatap, Hegde '17].
- ▶ Construct signal *marginal* matrix: $\mathbf{M} = \frac{1}{m} \sum_{i=1}^m y_i^2 \mathbf{a}_i \mathbf{a}_i^\top$.
- ▶ The j^{th} signal coefficient can be estimated from the the diagonal term $M_{jj} = \frac{1}{m} \sum_{i=1}^m y_i^2 a_{ij}^2$, and the set of all M_{jj} 's can be calculated in $\mathcal{O}(mn)$ time.
- ▶ Approximate support estimate \hat{S} via approximate model projection algorithm (for eg. [Hegde et. al. '14] for tree sparsity) on the $\text{diag}(\mathbf{M})$.
- ▶ Spectral initialization on sub-matrix $\mathbf{M}_{\hat{S}}$.

Initialization

Intuition

- ▶ Diagonal elements of the expectation matrix $\mathbb{E}[\mathbf{M}]$ are given by $\mathbb{E}[M_{jj}] = \|\mathbf{x}^*\|^2 + 2x_j^{*2}$.
- ▶ The signal marginals M_{jj} corresponding to $j \in S$ are larger, in *expectation*, than those corresponding to $j \in S^c$, where $S \in \mathbb{M}_S$.

Experimental validation

Ground truth

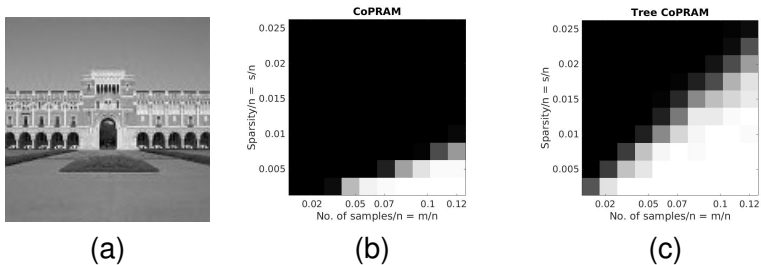


Figure: (a) Image considered for simulations, resized to 32×32 and 64×64 pixels, considered to be sparse in Haar basis. Phase transition diagrams for (b) CoPRAM (c) Tree CoPRAM on signal of dimension $n = 4096$.

Simulation results

Phase transitions

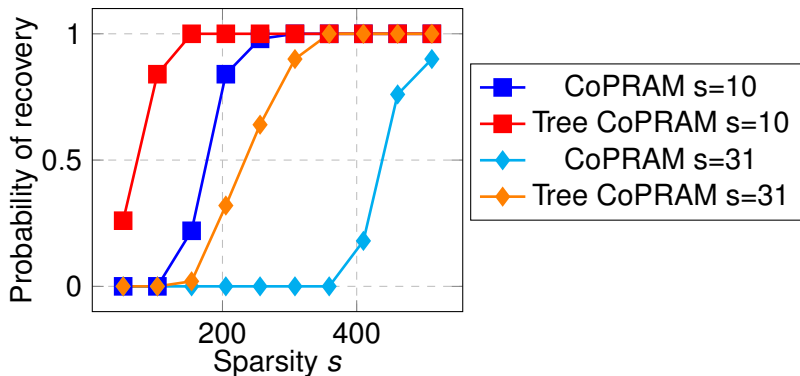


Figure: Phase transitions for CoPRAM and Tree CoPRAM for sparsities $s=10$ and $s=31$ on an $n=1024$ dimensional signal.

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- ▶ Sample optimal convergence analysis.

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Open questions:

- ▶ Theoretical guarantees on initialization.

Questions?

Interested in knowing more?
Check our project website:



<https://gaurijagatap.github.io/phase-retrieval-of-structured-signals/>